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Percolation under a spiralling constraint

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Abstract. A new type of percolation process, spiral percolation, is considered in which each step of a percolation path proceeds either straight or in a specific rotational direction. Monte Carlo simulation, assuming a finite-size scaling hypothesis, has been performed on the square lattice to determine the spiral percolation threshold p_s and the correlation length exponent ν_s . The values obtained are $p_s = 0.730 \pm 0.004$ and $\nu_s = 1.404 \pm 0.012$.

Percolation is one of the most widely studied phenomena in recent times. Percolation theory has extensive applications and attempts are still being made to understand various aspects of the problem (Stauffer 1985). One such aspect relates to the effect of external constraints on the percolation process. The most well known example of this is that of directed percolation (see Kinzel (1983)) where a global directional bias confines the percolating cluster to a narrow cone with axis along the preferred direction. This leads to anisotropic scaling and direction-dependent critical behaviour. The directed percolation problem is of considerable interest because it not only describes some real physical situations but also belongs to a universality class different from that for ordinary percolation. In this paper, we study the effect of another type of constraint, namely the spiralling constraint, on the percolation process. The spiralling constraint is effective if rotational force fields act on the system under consideration. The effect of such a constraint has already been studied in the case of self-avoiding walks (Privman 1983, Blöte and Hilhorst 1984, Guttmann and Wormald 1984) and lattice animals (Bose and Ray 1987) where significant changes in the walk or animal statistics have been found. In the following, we first describe what we call the spiral percolation problem. We then present results of a Monte Carlo simulation study of the spiral site percolation problem on a square lattice. The results include the values of the percolation threshold probability p_s for spiral percolation and the correlation length exponent ν_s with which the correlation length ξ diverges as the site occupation probability $p \rightarrow p_s$.

We consider site spiral percolation on a square lattice in which the sites are randomly occupied with probability p . As p is increased towards p_c , the percolation threshold for ordinary percolation, the incipient infinite cluster of connected sites starts making its appearance. This cluster of sites is, however, not connected if the spiralling constraint is taken into account. Under this constraint, two points i and j of the cluster are said to be connected if there is at least one spiral path between them. In the spiral connecting path, each step is executed either in the direction of the preceding step or turns in a specific rotational direction, say clockwise. Figure 1 gives an example of such connectivity, the bonds are labelled with arrow directions to trace out the spiral connections.

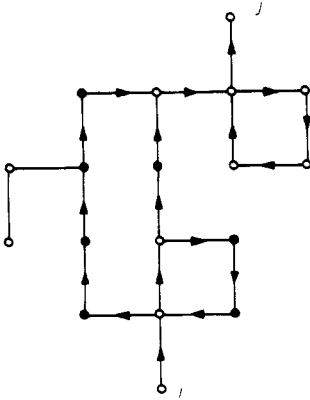


Figure 1. An example of spiral site percolation on a square lattice. The arrows on the bonds indicate the allowed spiral directions of flow from site i . The open circles indicate singly-connected sites, and the full circles multiply-connected sites. The sites connected by the unlabelled bonds are not spirally connected to site i .

The unlabelled bonds do not contribute to connectivity from site i . As the site occupation probability p is increased beyond p_c and reaches a threshold value p_s , it is to be expected that at this value spiral connection across the infinite cluster is just established. The value p_s is then identified with the percolation threshold probability for spiral percolation. For a clarification of the nature of spiral connection one should note the following: for a configuration of occupied sites there may be a spiral path from a site i to a site j but not from j to i (see figure 1). Also, if spiral connections exist between sites i and j and j and some other site k , it is not guaranteed that a spiral connection is present between i and k . In short, spiral connectedness is neither symmetric nor transitive. Consider a finite cluster (as in figure 1) in the configuration of occupied sites. Each site within the cluster is not spirally connected to every other one. One can only speak of the set $\{S_i\}$ of sites that can be reached spirally from a given site i . For spiral connection across the infinite cluster, there is at least one site i on the lower lattice edge whose set $\{S_i\}$ contains a site j on the upper lattice edge.

The distinguishing feature of the spirally-connected infinite cluster is the presence of a large number of loops. The loops constitute an essential feature since without them the infinite cluster will not be able to grow in all directions. Because of the loops, the percolating paths are much more tortuous than in the case of ordinary percolation. The spiral paths across the infinite cluster consist of both singly-connected and multiply-connected sites, a singly-connected site (open circles in figure 1) if removed breaks connection between points i and j whereas removal of a multiply-connected site (full circles in figure 1) does not destroy connectivity. Also, loop sites which are singly-connected in the case of spiral percolation become multiply-connected if the spiralling constraint is removed. However, as in the case of ordinary percolation (Coniglio 1981), the number of singly-connected sites or bonds (for the bond percolation problem) in spiral percolation diverges, as $p \rightarrow p_s$, with an exponent of exact value 1. The proof of this rests on the identity (Coniglio 1981, Sykes 1984) $p(dP_{ij}/dp) = L_{ij}$, where P_{ij} is the probability that sites i and j are connected and L_{ij} is the average number of singly-connected sites or bonds between i and j . The identity holds for any dimension and remains true when i and j are connected in the ordinary, directed or even in the

spiral sense. The rest of the proof is straightforward (see Coniglio (1981) for details) and is valid for both directed and spiral percolation.

For our Monte Carlo simulation study, we start with a finite square lattice of size $L \times L$. The spiral percolation threshold $p_s(L)$ is determined by the binary search method (Hoshen and Kopelman 1976). To begin with, the lattice sites are occupied with a certain probability p_0 with the help of a random number generator. It is then checked whether there is a spiral connection from at least one site of the bottommost row of the lattice to at least one site of the topmost row. In brief, the checking for spiral connection is done in the following way. At any stage of checking, there are some sites which are 'active', i.e. those sites which have already been checked to be connected spirally to the site at the bottommost row and from which further spiral connections are to be examined. For any of these 'active' sites, say, A, information is available as to the directions along which it has been visited from its preceding sites. We next examine the neighbouring occupied sites of A which can be connected spirally to A. This is done for all the 'active' sites. For the new sites thus obtained, we store information as to the directions in which they have been visited from the 'active' sites. In the next stage of checking, these new sites are to be considered as 'active' and the process continues. A site can be visited only once from each different direction and information about all such visits is kept stored. Hence, if a site is revisited at some later stage of checking, the site becomes 'active' only if it is visited in a direction along which it has not been visited previously. The checking process ends in the case of either of two situations: if we reach the topmost row of the lattice, the lattice is said to be spirally connected, otherwise, the paths end up within the lattice with no available sites for further spiral connection. The lattice is then not connected. If there is connection (no connection) p_0 is decreased (increased) by a small amount. The same random number sequence is then used to get estimates $p_1(L)$ and $p_2(L)$ which bound an interval containing the true threshold value $p(L)$. By successive binary chopping of this interval one determines $p_s(L)$ with the accuracy of 0.004. The whole process is then repeated N times ($N \times L \times L$ is of the order of 10^6) using different random number sequences. The average value $\langle p_s(L) \rangle$ of all the estimates obtained is taken as an estimate for the percolation threshold. The spread in the estimates is related to the correlation length exponent ν_s through the finite-size scaling formula (Levinshtein *et al* 1976, Reynolds *et al* 1980)

$$\Delta(L) = (\langle p_s^2(L) \rangle - \langle p_s(L) \rangle^2)^{1/2} L^{-1/\nu_s}, \quad (1)$$

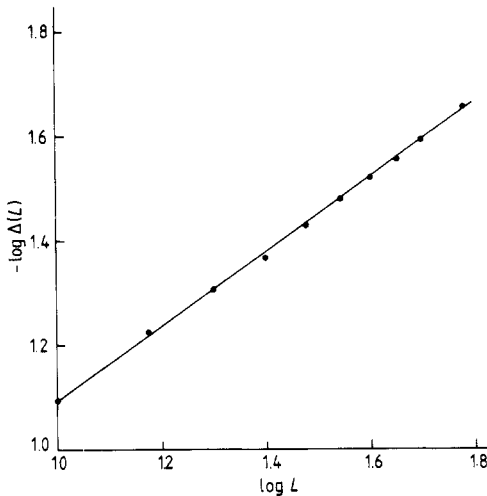
We have obtained values for $\langle p_s(L) \rangle$ and $\Delta(L)$ for lattices of size $L \times L$ with L ranging from 10 to 60, as listed in table 1. In figure 2 we have plotted $\log(\Delta^{-1}(L))$ against $\log L$, which is a straight line. The slope of this straight line gives the reciprocal of the correlation length exponent, from which we obtain $\nu_s = 1.404 \pm 0.012$. For ordinary percolation, the correlation length exponent $\nu_0 = \frac{4}{3}$. Again, if the finite-size scaling hypothesis is valid, then one can write down the relation

$$|p_s(\infty) - \langle p_s(L) \rangle| \sim L^{-1/\nu_s}, \quad (2)$$

where $p_s(\infty)$ is the percolation threshold probability in the limit of an infinitely large lattice. In figure 3, we have plotted $\langle p_s(L) \rangle$ against L^{-1/ν_s} using the value already obtained for ν_s . The plot is almost a straight line and we obtain the value of $p_s(\infty) = 0.730 \pm 0.004$. The value p_s is greater than the value p_c of the percolation threshold in case of ordinary percolation. This is to be expected since, as explained

Table 1. Monte Carlo results of $\langle p_s(L) \rangle$ and $\Delta(L)$ for all $L \times L$ lattices and over N realisations.

L	N	$\langle p_s(L) \rangle$	$\Delta(L)$
10	10 000	0.697 766	0.080 879
15	5 000	0.713 385	0.059 512
20	2 500	0.716 923	0.049 403
25	2 000	0.721 589	0.041 851
30	2 000	0.715 611	0.037 098
35	1 500	0.718 273	0.033 117
40	1 500	0.717 412	0.030 244
45	1 000	0.721 491	0.037 796
50	800	0.720 238	0.025 623
60	600	0.719 775	0.022 134

**Figure 2.** A plot of $-\log \Delta(L)$ against $\log L$ for square lattices of size $L \times L$. The slope of the straight line gives $\nu_s = 1.404 \pm 0.012$.

before, any connection between sites i and j of the infinite cluster does not guarantee spiral connection between them but the reverse is always true.

The difference in the values of the correlation length exponents ν_s and ν_0 for spiral and ordinary percolation suggests that percolation under a spiralling constraint may belong to a new universality class. To make a definite statement, much larger lattice sizes should be considered as there might be considerable deviations from the results obtained due to systematic (size-dependent) contributions. Such systematic errors have not been taken into account in the error bars quoted for the values of ν_s and p_s . On the other hand, a finite-size scaling hypothesis for ordinary percolation seems to apply in the case of spiral percolation also. As has been mentioned before, the spiralling constraint has a non-trivial effect on both self-avoiding walk and lattice animal problems. For spiral lattice site animals, it has been shown (Bose *et al* 1987) that animals and trees (animals without loops) belong to different universality classes, i.e. loops have a non-trivial effect on the animal statistics. This result is contrary to the result

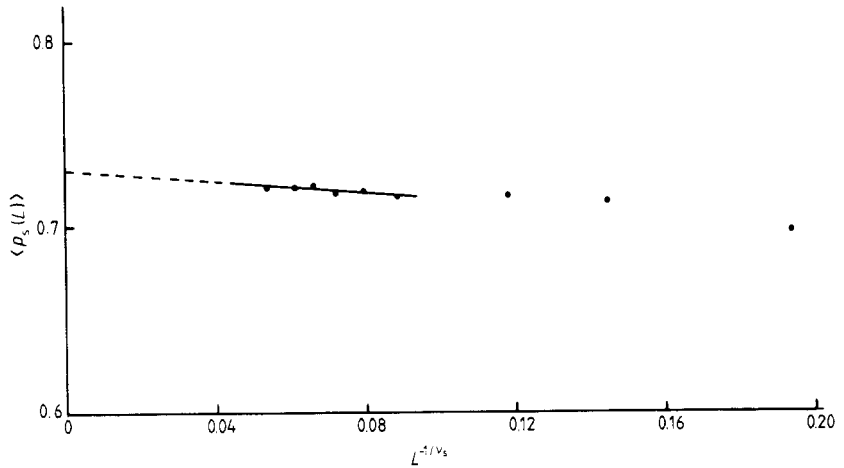


Figure 3. A plot of $\langle p_s(L) \rangle$ against L^{-1/ν_s} . The intercept on the vertical axis gives $p_s = 0.730 \pm 0.004$.

obtained in the case of ordinary lattice animals (Gaunt *et al* 1982, Duarte 1985). It is to be expected that loops also play a vital role in the spiral percolation process and a study of the loop-size distribution function and other loop properties should be of definite interest. A more detailed study of the spiral percolation problem is in progress and the results will be reported elsewhere.

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